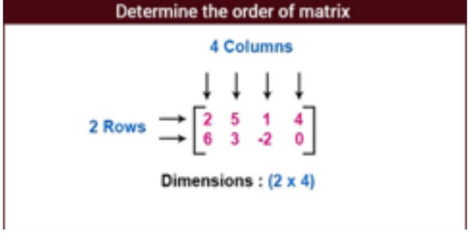
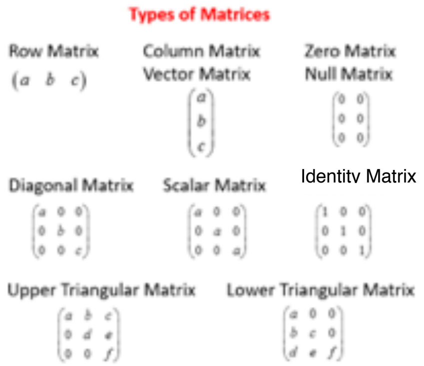
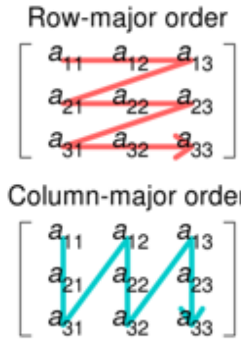


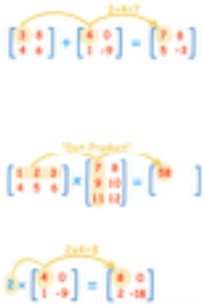
Only square matrices (N x N) can be squared (²)
Matrices can't be divided

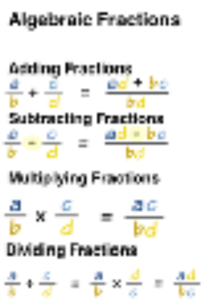
In order to multiply matrices the column number of the first matrix must equal the row number of the second. The row number of the first matrix and the column number of the second will be the dimensions/order of the new matrix
Yes
(A x B) x (B x C) = A x C
No
(A x B) x (C x D) = CBD
In order to add or subtract matrices the they must have the same dimensions/order. If they can be added or subtracted they will equal a matrix of the same order
Yes
(A x B) + (A x B) = A x B
(A x B) - (A x B) = A x B
No
(A x B) + (C x D) = CBD
(A x B) - (C x D) = CBD



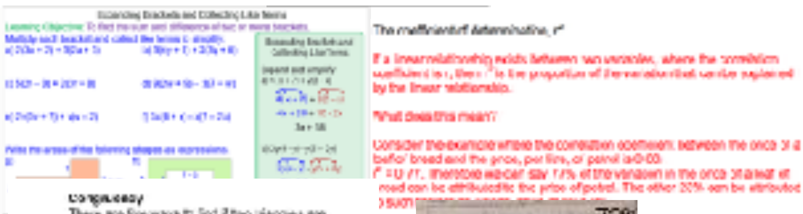


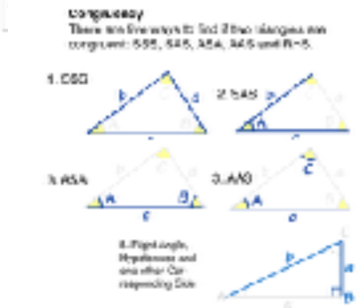
Common words
Communative = B + A = A + B
Associative = A + (B + C) = (A + B) + C

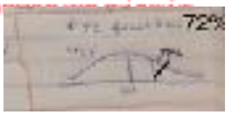




Walk - a sequence in which you can reach each vertex from the previous e.g. ABDCAB
Path - a walk that uses no repeat edges or vertices e.g. ADBC
Trail - a walk that uses no repeat edges, can repeat vertices e.g. ABDCB
Eulerian trail: a trail that uses no repeat edges but visits every vertex e.g. ABDC
Eulerian circuit: a trail that uses no repeat edges, visits every vertex and finishes at the same vertex it started at (must have no odd vertices)
Semi-Eulerian: a trail that uses no repeat edges, visits every vertex but finishes at a different vertex to the one it started at
Hamiltonian path: A path that visits every vertex with no repeat edges or vertices, edges can remain unused e.g. ADBC
Hamiltonian cycle: A path that visits every vertex with no repeat edges or vertices and finishes at the vertex it started. edges can remain unused e.g. ADBCA
Semi Hamiltonian: A path that visits every vertex with no repeat edges or vertices and finishes at the a different vertex to the one it started. edges can remain unused e.g. ADBC
Euler's rule: V+F=E+2



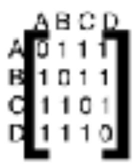




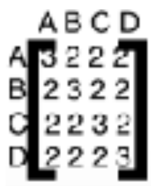
An item is discounted by 15% and it's new price is $480. What was the original price (y)?
y x 0.85 = 480
y = 480/0.85
y = $564.71

Categorical Variables
Nominal - no order is suggested
Ordinal - order is suggested
Numerical Variables
Continuous - measured (e.g. 2.5 kg, 8.7km)
Discrete - Counted (e.g. 2 people, 3 dogs)

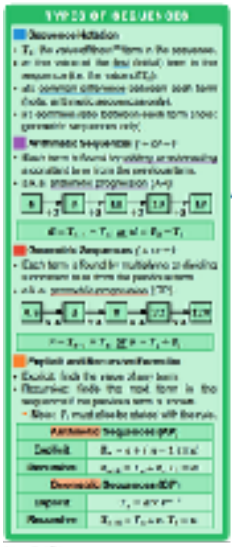
When Scores are normally distributed
1 standard deviation from the mean = 68.3% of scores
2 standard deviation from the mean = 95.4% of scores
3 standard deviation from the mean = 99.7% of scores

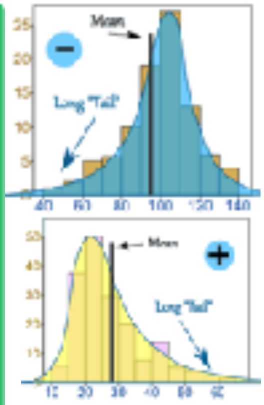


Adjacency matrix

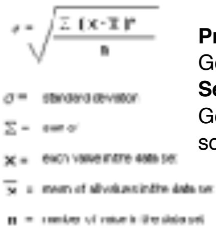


Adjacency matrix² = 2 route matrix (ways to get from one vertex to another using two edges





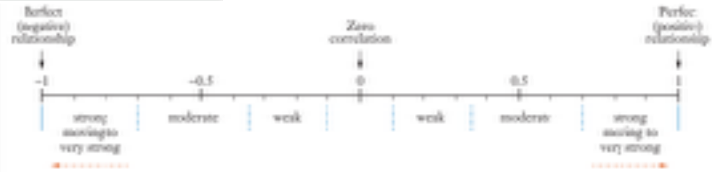
For a row or column percentage go with the explanatory variable
E.g. if the explanatory value is on the y axis of the table use row percentages. If it is on the x axis use column percentages



Primary Data
Got yourself
Secondary data
Got from someone/ somewhere

Standardised score =

<u>Raw Score - Mean</u>
Standard deviation



Pearson's Correlation Coefficient (r)
The value r such that $-1 \leq r \leq 1$ measures the direction and strength of a linear relationship between two variables.

Coefficient of Determination (r^2)
The value r^2 such that $0 \leq r^2 \leq 1$ shows the percentage of the variation in the response variable with the variation in the explanatory variable. It shows what percent of the data that is the closest to the line of best fit (i.e. if $r^2 = 0.85$, then 85% of the data is close to the line of best fit). Also, r^2 is equal to Pearson's Correlation Coefficient squared.

Least-Squares Line/Line of Best Fit ($y = ax + b$)
A linear equation that summarises the relationship between two variables where a is the gradient of the line (calculated by $a = $ rise/run) and b is the y-intercept.

Compound Interest Recurrence Relation

$$A_{n+1} = \left(1 + \frac{i}{n}\right)A_n + r, \quad A_0 = P$$

i: interest rate (as a decimal)
n: number of times interest is compounded per year
r: regular payments (for investments, r is positive and for loans and annuities, r is negative)
P: principal (initial amount)

Response and explanatory variables

Response Variable	Explanatory Variable
Dependent Variable	Independent Variable
Vertical Axis (y-axis)	Horizontal Axis (x-axis)

Explanatory Variable causes the Response Variable (e.g. being immunised causes resistance to disease or number of books read causes reading speed to be low, medium or high)

Compound Interest Formulas

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad I = A - P$$

A: amount (principal plus interest)
P: principal (starting amount)
I: total amount of interest
r: annual interest rate (as a decimal)
n: number of times interest is compounded per year
t: time in years

Effective Annual Rate
Effective annual rate of interest converts i% p.a. compounding n times per year to R% p.a. compounding annually.

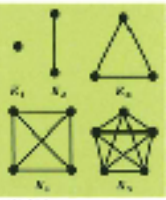
$$i_{effective} = \left(1 + \frac{i}{n}\right)^n - 1$$

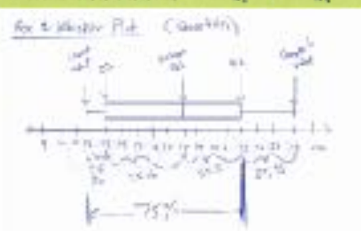
$i_{effective}$: effective annual rate of interest (as a decimal)
i: annual interest rate (as a decimal)
n: number of times per year that interest is compounded

Complete Graph (K_n)
A graph with n vertices in which every vertex is connected to every other vertex by one edge.

Edges in $K_n = n(n-1)/2$

Number of edges follow the triangular number sequence: 0, 1, 3, 6, 10, 15, 21, 28... (i.e. difference between each number is 1, 2, 3, 4, 5, 6, 7...)





The coefficient of determination, r^2.

If a linear relationship exists between two variables, where the correlation coefficient is r, then r^2 is the proportion of the variation that can be explained by the linear relationship.

What does this mean?

Consider the example where the correlation coefficient between the price of a loaf of bread and the price, per litre, of petrol is 0·88.
r^2 = 0·77. Therefore we can say 77% of the variation in the price of a loaf of bread can be attributed to the price of petrol. The other 23% can be attributed to such factors as wages, price of flour etc.

3, 17, 73, 297, 1193...
this can be defined in the form

$$T_{n+1} = aT_n + b$$

$T_2 = aT_1 + b$ and $T_3 = aT_2 + b$
$17 = 3a + b$ $\quad$ $73 = 17a + b$

If the 10th term of a geometric progression is 98415 and the 13th term is 2657205, write a recursive and explicit formula
98415 __ __ 2657205 — 98415 x r x r x r = 2657205 — 98415 x r^3 = 2657205
solve it — r = 3 — T1 = 98415 / 9 — T1 = 5
recursive -> Tn+1 = 3Tn, T1 = 5 explicit rule = Tn = 3 x 5 (n-1)

Hungarian Algorithm Steps
1. If the question asks for maximising take the largest number and subtract each number from it (as below) then minimise

482 437 512
421 399 432
502 407 518

→

518-482 518-437 518-512
518-421 518-399 518-432
518-502 518-407 518-518

→

36 81 6
97 119 86
16 111 0

2. Subtract the smallest number in each row from all other numbers in the row

36 81 6
97 119 86
16 111 0

→

36-6 81-6 6-6
97-86 119-86 86-86
16-0 111-0 0-0

→

30 75 0
11 33 0
16 111 0

3. Subtract the smallest number in each column from all other numbers in the column

30 75 0
11 33 0
16 111 0

→

30-11 75-33 0-0
11-11 33-33 0-0
16-11 111-33 0-0

→

19 42 0
0 0 0
5 78 0

4. use minimal lines to cross out all zeroes. Anything crossed out, do nothing to, where the lines intersect add the smallest number in the matrix, where the number isn't crossed out, subtract the smallest number. Repeat this step until the number of lines used = the amount of rows/columns in the matrix (e.g. 3x3 matrix needs 3 lines used)

19 42 0
0 0 0
5 78 0

→

19-5 42-5 0
0 0 0+5
5-5 78-5 0

→

14 37 0
0 0 5
0 73 0

5. highlight the zeroes ensuring there is one highlighted in each row and column

14 37 0
0 0 5
0 73 0

6. Add the original numbers from the positions

482 437 512
421 399 432
502 407 518

Max Profit = 512+399+502 = 1413

Prim's Algorithm Steps

1. Pick a starting Vertex (e.g. A). Put an arrow above column A and cross out row A

	S	A	B	C	D	E	T
S	-	2	5	5	-	-	-
~~A~~	~~2~~	~~-~~	~~2~~	~~-~~	~~7~~	~~-~~	~~-~~
B	5	2	-	2	5	3	-
C	5	-	2	-	-	4	-
D	-	7	5	-	1	5	-
E	-	-	3	4	1	-	7
T	-	-	-	-	5	7	-

2.look for the smallest value in column A and highlight

	S	A	B	C	D	E	T
S	-	2	5	5	-	-	-
~~A~~	~~2~~	~~-~~	~~2~~	~~-~~	~~7~~	~~-~~	~~-~~
B	5	2	-	2	5	3	-
C	5	-	2	-	-	4	-
D	-	7	5	-	1	5	-
E	-	-	3	4	1	-	7
T	-	-	-	-	5	7	-

3. Put an arrow above column B and cross row B

	S	A	B	C	D	E	T
S	-	2	5	5	-	-	-
~~A~~	~~2~~	~~-~~	~~2~~	~~-~~	~~7~~	~~-~~	~~-~~
~~B~~	~~5~~	~~2~~	~~-~~	~~2~~	~~5~~	~~3~~	~~-~~
C	5	-	2	-	-	4	-
D	-	7	5	-	1	5	-
E	-	-	3	4	1	-	7
T	-	-	-	-	5	7	-

4. Look for the smallest values in A and B

	S	A	B	C	D	E	T
S	-	2	5	5	-	-	-
~~A~~	~~2~~	~~-~~	~~2~~	~~-~~	~~7~~	~~-~~	~~-~~
~~B~~	~~5~~	~~2~~	~~-~~	~~2~~	~~5~~	~~3~~	~~-~~
C	5	-	2	-	-	4	-
D	-	7	5	-	1	5	-
E	-	-	3	4	1	-	7
T	-	-	-	-	5	7	-

5. repeat steps until all vertices have been chosen

	S	A	B	C	D	E	T
S	-	2	5	5	-	-	-
~~A~~	~~2~~	~~-~~	~~2~~	~~-~~	~~7~~	~~-~~	~~-~~
~~B~~	~~5~~	~~2~~	~~-~~	~~2~~	~~5~~	~~3~~	~~-~~
C	5	-	2	-	-	4	-
D	-	7	5	-	1	5	-
E	-	-	3	4	1	-	7
T	-	-	-	-	5	7	-

TYPES OF VARIABLES

Response and Explanatory Variables

- Response Variable (*RV*)
  - Also known as the dependent variable.
  - Plotted on the vertical axis ($y-axis$).
- Explanatory Variable (*EV*)
  - Also known as the independent variable.
  - Plotted on the horizontal axis ($x-axis$).

The Response Variable (*RV*) depends on the Explanatory Variable (*EV*)

Examples of RV's with Matching EV's

- The RV, weight loss (*kg*), depends on the EV, time spent dieting (*days*).
- The RV, wage (*dollars*), depends on the EV, time spent working (*hours*).
- The RV, heart rate (*bpm*), depends on the EV, caffeine consumption (*mg*).

FINANCIAL CALCULATOR

Compound Interest Financial Calculator

(Q1) Jackson borrows $20,000 at 12% p.a. compounding monthly. He pays $350 every month to pay off the loan. How much would he still owe after 5 years of payments?

Type	Loan	PMT	350
N	$5 \times 12 = 60$	FV	$-\$7749.55$
I%	12	P/Y	12
PV	20000	C/Y	12

DESEASONALISING DATA

Deseasonalising Data

- Smoothing the data to reduce the presence of seasonal fluctuations (e.g. reduce the fluctuations in apple picking between summer and winter months of each year).

Step 1	Calculate the average of each of the non-seasons. Non-seasons are typically years, months or weeks.
Step 2	Divide each of the original data values respectively by the average of the non-seasons found in Step 1.
Step 3	Finding Seasonal Indices. Calculate the average of each of the seasons found in Step 2. Seasons are commonly time periods, days or seasons (summer, winter, autumn, spring) of the year.
Step 4	Deseasonalising the Data. Divide each of the original data values respectively by the seasonal indices found in Step 3.

Deseasonalising Data Example

(Q1a) Deseasonalise the following data set

Yr/Pd	Period 1	Period 2	Period 3
Year 1	411	648	599
Year 2	[illegible]	[illegible]	741

- Step 1: Average non-seasons (i.e. years).

Average Year 1	$\frac{411 + 648 + 599}{3} = 586$
Average Year 2	$\frac{[illegible] + [illegible] + 741}{3} = 635$

- Step 2: Divide original data by Step 1.

Yr/Pd	Period 1	Period 2	Period 3
Year 1	$\frac{411}{586} = [illegible]$	$\frac{648}{586} = [illegible]$	$\frac{699}{586} = [illegible]$
Year 2	$\frac{532}{635} = [illegible]$	$\frac{632}{635} = [illegible]$	$\frac{532}{635} = [illegible]$

- Step 3: Find seasonal index by averaging seasons in Step 2 (i.e. periods)

Average Period 1	[illegible]
Average Period 2	$\frac{[illegible]}{2} = 1.0585$
Average Period 3	$\frac{[illegible]}{2} = 1.1796$

- Step 4: Deseasonalise data by dividing the original data respectively by Step 3

Yr/Pd	Period 1	Period 2	Period 3
Year 1	$\frac{411}{[illegible]} = 534$	$\frac{648}{[illegible]} = 617$	$\frac{699}{[illegible]} = 502$
Year 2	$\frac{[illegible]}{[illegible]} = 691$	$\frac{632}{[illegible]} = 602$	$\frac{[illegible]}{[illegible]} = 620$

(Q1b) Graph the deseasonalised data.